A novel omnidirectional wheel based on Reuleaux-triangles

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Abstract. This paper discusses the mechanical design and simulation of a novel omnidirectional wheel based on Reuleaux-triangles. The main feature of our omniwheel is that the point of contact of the wheel with the floor is always kept at the same distance from the center of rotation by mechanical means. This produces smooth translational movement on a flat surface, even when the profile of the complete wheel assembly has gaps between the passive rollers. The grip of the wheel with the floor is also improved. The design described in this paper is ideal for hard surfaces, and can be scaled to fit small or large vehicles. This is the first design for an omnidirectional wheel without circular profile, yet capable of rolling smoothly on a hard surface.

1 Motivation and Reuleaux triangles

It has been thought for many years, that the only way of providing a smooth rolling effect when using omnidirectional wheels[1] with gaps between rollers is: a) by stacking two wheels on the same axis, producing a combined circular profile; b) by using several synchronized wheels which combine in order to support the vehicle keeping the distance to the floor constant; or c) by using spheres or quasi-spheres as wheels. There is a fourth alternative, which is to design omniwheels with a circular profile, in which the gaps are almost closed by using two kinds of rollers alternatively, as in [3]. Long and short rollers alternate on the periphery of the wheel. In this paper we show for the first time that it is possible to build an omnidirectional wheel *with gaps* between the transversal rollers, that is without circular profile, which is nevertheless able to drive smoothly.

A so-called Reuleaux triangle (named after the German engineer Franz Reuleaux, who was a professor of mechanical engineering at the Technical University of

[1] The very first omnidirectional wheel was patented in 1919 by J. Grabowiecki in the US [1]. Bengt Ilon patented another in 1973 [2].

Berlin) is a geometric shape whose width remains constant during rotation. This means that two parallel lines in contact with the shape's boundary stay at the same distance independently of the shape's orientation. The simplest shape of constant width is a circle. Other geometric figures can be modified to have constant width. Fig. 1 shows how a Reuleaux triangle is constructed. Starting from an equilateral triangle of side length l, constant width is achieved adding circular arcs with radius l to each of the triangle's sides. The center of each arc is placed at the corner opposite the respective side. Reuleaux first mentioned these triangles in 1876 [4].

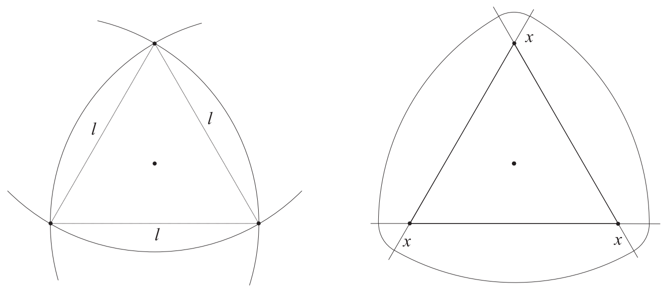

Fig. 1. Construction of the Reuleaux triangle

The shape's area can be increased by extending the triangle's sides beyond the corners by a distance x (Fig 1, right). The arcs' radii become then $l + x$. The gaps outside the original triangle and between two crossing sides are closed by circular arcs of radius x. In the following, the smaller arcs of radius x will be referred to as l_x-sections, and the larger arcs as l_r-sections.

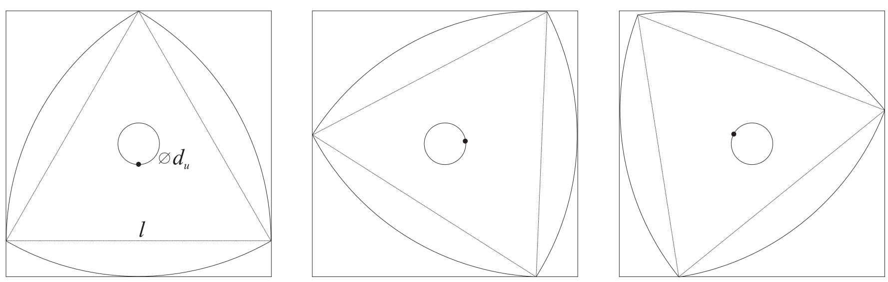

Fig. 2. The Reuleaux triangle rotating inside a square

The characteristics of a Reuleaux triangle allows such shape to rotate inside a square, as a circle also can (Fig. 2, see [5] for an explanation). Notice that the triangle's center *does not* remain in the same place while rotating the triangle, but moves along a curve consisting of four elliptical arcs. For the wheel design presented in this paper, this curve is approximated by a circle[2]. The circle's diameter d_u is obtained using the following formula:

$$d_u = l \cdot (\frac{4}{3} \cdot \cos(\pi/6) - 1)$$

obtained from elementary geometrical considerations. This diameter *remains the same* for a given l, independent of the chosen extension of length x. Therefore, the principal movement can be obtained from considering simple Reuleaux-triangles (that is, triangles for which $x = 0$) instead of extended triangles. This fact is important since the wheel design presented here makes use of extended Reuleaux-triangles.

2 Wheel design

The omnidirectional wheel proposed in this paper consists basically of the following parts:

1. Two discs based on Reuleaux-triangles, each carrying three passive wheels.
2. A gear connecting these discs, which allows the transmission of rotation between both.
3. An excenter which holds both discs and the gear.

In our wheel, a Reuleaux-triangle of enhanced area is used for the shape of the component which carries each group of three passive wheels (Fig. 3). The shape of the passive wheels' profile is determined by the l_x-sections described above. The l_r-sections remain empty, except for small supporting structures.

The structures do not allow the passive wheels to lose contact with the ground as long as the next passive wheel has not reached the ground yet. The path described by such passive wheels is explained below.

The passive wheel is built by using the l_x-section shape as the profile of a roller. Note that the position of the center axis of this roller (which is actually the

[2] The curve is nearly the superellipse $x^{2.36} + y^{2.36} = c$, where c is a constant [5].

passive wheel's axis) may be chosen arbitrarily. Thus it is possible to construct passive wheels of different sizes.

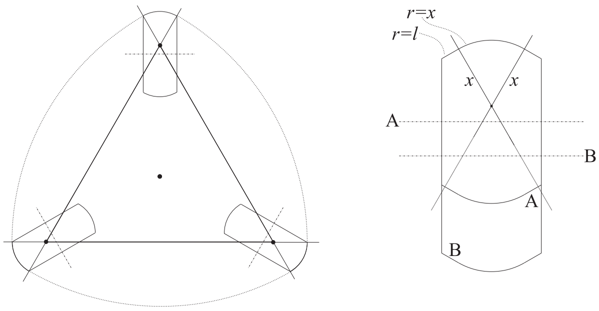

Fig. 3. Profile of the Reuleaux wheel (left). The wheel has three passive rollers. The rollers can have a smaller radius (centerline *A*), or a larger radius (centerline *B*). In each case, the effect is the same.

Before describing the remaining parts, it is necessary that the reader understands the principle behind the functionality of this omnidirectional wheel.

As described above, a Reuleaux-triangle can be rotated inside a square [5]. The square's sides are touched alternately by the triangle's corners and the circular arcs (in the extendend triangle by the l_x-section and the l_r-section). Imagine one of the square's sides to be the ground over which the Reuleaux discs roll, with the passive wheels touching the ground (remember that the passive wheels' profile matches the curve that describes the l_x-section). Each disc has contact with the ground only part of the time, because when the l_x-section loses contact with one side, the l_r-section reaches that side. Remember that the l_r-section-surface is *empty* in the Reuleaux discs.

To guarantee ground contact the entire time, a second Reuleaux disc is added to the wheel design, describing the exact same path of movement, but with a phase shift. The phase shift corresponds to exactly one half of a rotation of the Reuleaux-triangles' center around the displacement path of diameter d_u, as illustrated in Fig. 4 (one triangle looks as "flipped" by vertical reflection).

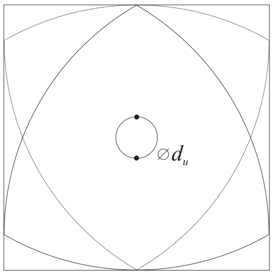

Fig. 4. Two Reuleaux discs shifted in phase

This ensures that the Reuleaux discs' passive wheels always touch the ground, alternating from one disc to the other. A simulation of a working omnidirectional wheel following this principle is shown in Fig. 5. The advantage of this wheel design becomes clear immediately: The passive wheels describe a purely linear path along the ground, preventing any up- and down-movement. On the contrary, since conventional omnidirectional wheels are shaped as n-side polygons (see Fig. **??**, left diagram), this leads to an up-and-down movement of the robot, which is lifted up every time a passive wheel touches the ground . At high velocities, the entire wheel might lose ground contact, making it difficult to drive accurately (as often happens during RoboCup competitions.

3 Design of excenter and gears

In order to achieve the circular movement of the center of each Reuleaux disc an excenter is needed. This excenter is held in place by the motor axis to which the torque is applied in order to drive the omnidirectional wheel. The first Reuleaux disc center axis is placed at a distance $\frac{1}{2}d_u$ of the motor axis (black rod in Fig. 6). When the excenter is rotated, the Reuleaux disc's center describes the desired curve (a circle of diameter d_u).

Additionally, for each complete rotation around the excenter, the Reuleaux disc itself needs to rotate by $-\frac{2}{3}\pi$ around its center. This combined movement is realized by two connected gear wheels. The smaller gear wheel, with the sprockets on the outside, is fixed to the Reuleaux disc. Its diameter must be $3 \cdot d_u$. Since this gear wheel rotates around the motor axis excentrically, a counterweight may be attached to the excenter to compensate the gear wheel's centrifugal force. The second larger gear wheel, with its sprockets on the inside, is fixed to the robot. Its diameter is $4 \cdot d_u$. Fig. 6 shows the complete configuration.

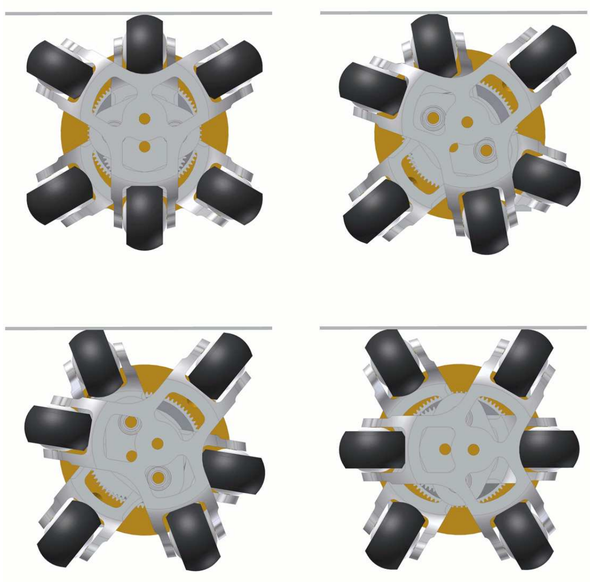

Fig. 5. Rotation of the wheel and relative displacement of the rollers. The wheel starts on the upper left and rolls counterclockwise. The passive roller on the ground loses contact only when the next roller has reached the ground-contact position. The combined movement looks as if the next roller "stretches a leg" before reaching ground.

Now that the first Reuleaux disc describes the desired movement, the second disc needs to be connected to imitate the exact same movement with a phase shift. To obtain a phase shift of $\frac{1}{3}\pi$, we just flip the Reuleaux structure by 180 degrees, as shown in Fig. 7. Building a gear for this task is not difficult: the second disc's center axis is placed at a distance $\frac{1}{2}d_u$ from the motor axis, as before, but in the opposite direction of the first disc's axis. In order to rotate the second disc, a transmission gear is constructed to provide a 1:1 transmission of rotation from the first to the second disc (Fig. 7).

4 Conclusions

This paper has presented a new design for an omnidirectional wheel based on Reuleaux triangles. We have shown that even when the complete profile of the wheel mount has "gaps" between the passive rollers, it is indeed possible to achieve smooth rotational movement. For this to occur, the wheel center rotates

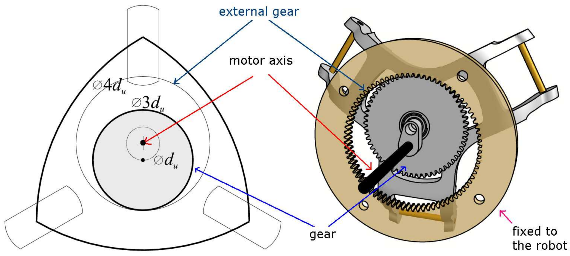

Fig. 6. Gear wheels and their diameter. When rotating around the gear fixed to the robot, the center of the Reuleaux disc describes a circle of radius d_u.

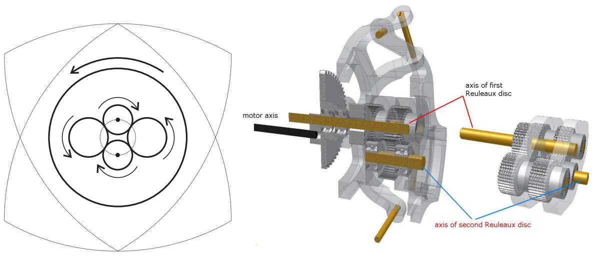

Fig. 7. Assembly and movement of the two Reuleaux discs. To the left a diagram, in the middle a cross section of the assembly, and to the right the transmission gear.

around an excenter. A conventional wheel with profile gaps resembles a polygon and a rotating polygon moves the wheel's center up and down – if nothing is done against that movement. The wheels used by almost all teams in the small-size league resemble such polygons. Mid-size teams have used also wheels with perpendicular passive rollers and gaps in the wheel profile. The wheels presented in this paper are an alternative solution.

We are aware that the wheel design described in this paper is more complex than other omnidirectional wheels and requires more mechanical parts. Nevertheless, the wheel presented here has some interesting theoretical properties. Firstly, its profile is not circular, correcting the long held misconception that only circular-profile omniwheels can roll smoothly. Secondly, this design allows a robot (or other vehicle) highly precise and controllable omnidirectional driving, even at high speeds. Wheel grip is optimal because driving is vibration free, and the individual passive wheels have contact with the ground over a long time. Modelling Mecano wheels is extremely complex (because of the angle at which the passive rollers are placed) [6]. Kinematic modeling of the Reuleaux wheels is much simpler.

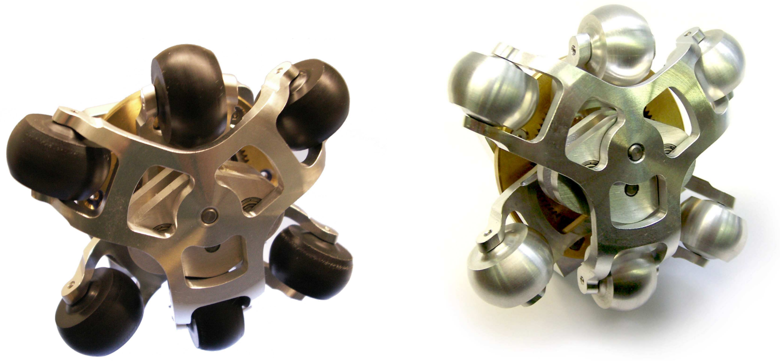

Fig. 8. Our first two Reuleaux omnidirectional wheel prototypes

Fig. 8 shows a ready-to-build sample design, including all the components described above and a photograph of the first prototype. The smooth rolling movement was validated in the computer before the prototype was built. Two Reuleaux omnidirectional wheels will be on display at the RoboCup 2006 competition in Bremen. It is a new attempt at reinventing the wheel, right, but the omnidirectional wheel.

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