An omnidirectional wheel based on Reuleaux-triangles

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Abstract. This paper discusses the mechanical design and simulation of a novel omnidirectional wheel based on Reuleaux-triangles. The main feature of our omniwheel is that the point of contact of the wheel with the floor is always at the same distance from the center of rotation. This produces smooth translational movement on a flat surface, even when the profile of the complete wheel assembly has gaps between the passive rollers. The grip of the wheel with the floor is also improved. The design described in this paper is ideal for hard surfaces, and can be scaled to fit small or large vehicles.

1 Omnidirectional wheels

Omnidirectional wheels have been used in robotics, in industry, and in logistics for many years. The main source for omnidirectional wheels are companies which produce them for omnidirectional transportation tables, such as those used by postal or package handling companies. Omnidirectional wheels have been used to build omnidirectional robots, specially in the RoboCup setting. An omnidirectional robot can drive along a straight line from point to point, while rotating along this line in order to arrive with the correct orientation. Omnidrectional wheels have also been used for wheel chairs, for service vehicles in airports, and many other applications.

The first omnidirectional wheels were developed by the Swedish inventor Bengt Ilon around 1973 [1]. Fig. 1 shows the design of the wheel and a fork lift built using them. The profile of the wheel is more or less round. The wheel is omnidirectional but transversal forces produce excessive friction in the axes of the small rollers. A clever alternative are "Killough rollers" [2], which are usually built using two truncated spheres. Such rollers were used by the Cornell RoboCup team in 2000, and were still in use until 2004 in RoboCup competitions. Many RoboCup teams use now self-built wheels or a variation of commercial wheels built for transport platforms, such as those shown in Fig. 2. Geometrically, such wheels are polygons with small transversal rollers at the polygon's corners (as can be seen in the left wheel). The wheels are made round by using a stack of two of them and avoiding any gap in the wheel profile (right wheel in Fig. 2). The point of contact changes from one wheel to the other as the complete wheel rolls.

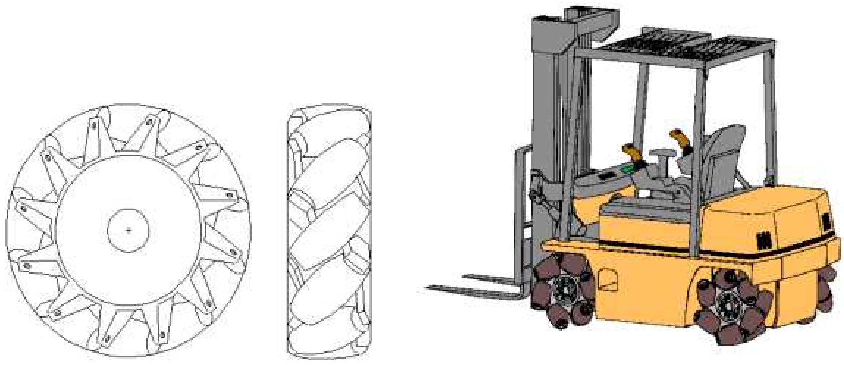

Fig. 1. Omnidirectional wheel and omnidirectional fork lift

Fig. 2. Segmented omnidirectional wheel and a commercial double wheel

Much effort has been spent on improving the Swedish or Mecanum wheels, as they are sometimes called [3]. The omniwheels can only roll smoothly if the profile of the complete wheel assembly is perfectly round, without gaps. However, as we show in the next sections, it is possible to build an omnidirectional wheel with transversal rollers with gaps between the rollers which is nevertheless able to drive smoothly.

2 Reuleaux triangles

A so-called Reuleaux triangle (named after the German engineer Franz Reuleaux, who was a professor of mechanical engineering at the Technical University of Berlin) is a geometric shape whose width remains constant during rotation. This means that two parallel lines in contact with the shape's boundary stay at the same distance independently of the shape's orientation. The simplest shape of constant width is a circle. Other geometric figures can be modified to have constant width. Fig. 3 shows how a Reuleaux triangle is constructed. Starting from an equilateral triangle of side length l, constant width is achieved adding circular arcs with radius l to each of the triangle's sides. The center of each arc is placed at the corner opposite the respective side. Reuleaux first mentioned this triangles in 1876 [4].

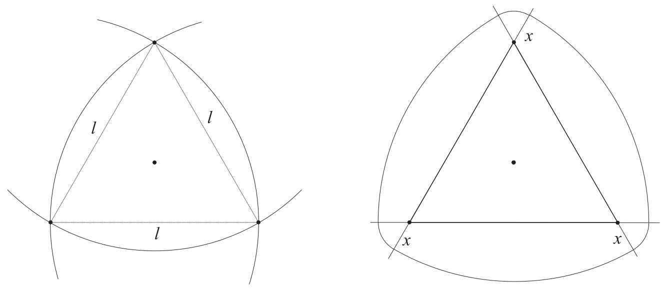

Fig. 3. Construction of the Reuleaux triangle

The shape's area can be increased by extending the triangle's sides beyond the corners by a distance x (Fig 3). The arcs' radii become then $l + x$. The gaps outside the original triangle and between two crossing sides are closed by circular arcs of radius x. In the following, the smaller arcs of radius x will be referred to as l_x-sections, and the larger arcs as l_r-sections.

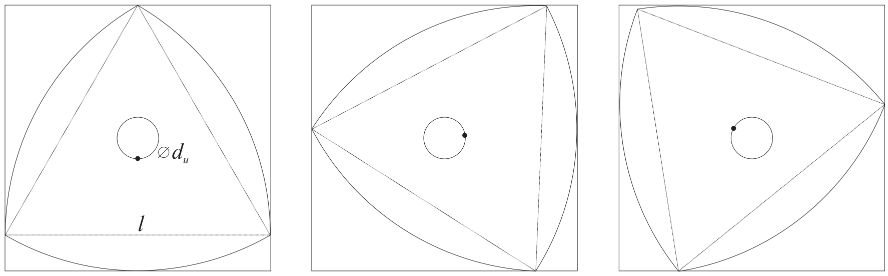

Fig. 4. The Reuleaux triangle rotating inside a square

The characteristics of a Reuleaux triangle allows such shape to rotate inside a square, as a circle also can (Fig. 4, see [5]). Notice that the triangle's center *does not* remain in the same place while rotating the triangle, but moves along a curve consisting of four elliptical arcs. For the wheel design presented in this paper, this curve is approximated by a circle. The circle's diameter d_u is obtained using the following formula:

$$d_u = l \cdot (\frac{4}{3} \cdot \cos(\pi/6) - 1)$$

obtained from elementary geometrical considerations. This diameter *remains the same* for a given l, independent of the chosen extension of length x. Therefore, the principal movement can be obtained from considering simple Reuleaux-triangles (that is, triangles for which $x = 0$) to extended triangles. This fact is important since the wheel design presented here makes use of extended Reuleaux-triangles.

3 Wheel design

The omnidirectional wheel proposed in this paper consists basically of the following parts:

1. Two discs based on Reuleaux-triangles, each carrying three passive wheels.
2. A gear connecting these discs, which allows the transmission of rotation between both.
3. An excenter which holds both discs and the gear.

3.1 Reuleaux discs

In our wheel, a Reuleaux-triangle of enhanced area is used for the shape of the component which carries each group of three passive wheels (Fig. 5). The shape of the passive wheels' profile is determined by the l_x-sections described above. The l_r-sections remain empty, except for small appendices.

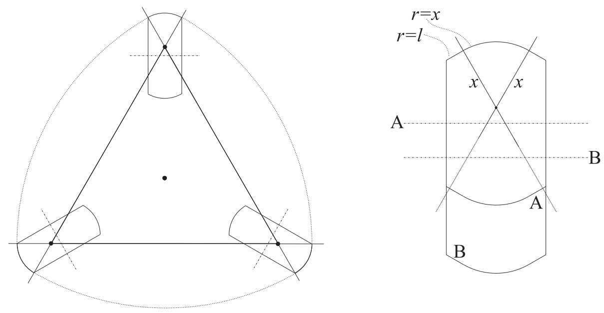

Fig. 5. Profile of the Reuleaux wheel

The appendices do not allow the passive wheels to lose contact with the ground as long as the next passive wheel has not reached the ground yet. The path described by such passive wheels is explained below.

The passive wheel is built by using the mentioned shape as the profile of a roller. Note that the position of the center axis of this roller (which is actually the passive wheel's axis) may be chosen arbitrarily. Thus it is possible to construct passive wheels of different sizes.

3.2 The wheel's movement

Before describing the remaining parts, it is necessary that the reader understands the principle behind the functionality of this omnidirectional wheel.

As described above, a Reuleaux-triangle can be rotated inside a square [5]. The square's sides are touched alternately by the triangle's corners and the circular arcs (in the extendend triangle by the l_x-section and the l_r-section). Imagine one of the square's sides to be the ground over which the Reuleaux discs roll, with the passive wheels touching the ground (remember that the passive wheels' profile matches the curve that describes the l_x-section). Each half-wheel has contact with the ground only part of the time, because when the l_x-section loses contact with one side, the l_r-section reaches that side. Remember that the l_r-section-surface is *empty* in the Reuleaux discs.

To guarantee ground contact the entire time, a second Reuleaux disc is added to the wheel design, describing the exact same path of movement, but with a phase shift. The phase shift corresponds to exactly one half of a rotation of the Reuleaux-triangles' center around the displacement path of diameter d_u, as illustrated in Fig. 6.

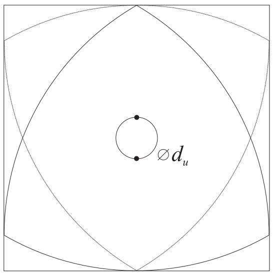

Fig. 6. Two Reuleaux discs shifted in phase

This ensures that the Reuleaux discs' passive wheels always touch the ground, alternating from one disc to the other. A simulation of a working omnidirectional wheel following this principle is shown in Fig. 7. The advantage of this wheel design becomes clear immediately: The passive wheels describe a purely linear path along the ground, preventing any up- and down-movement. Conventional omnidirectional wheels are shaped as n-side polygons, which leads to a movement where the robot is lifted up every time a passive wheel touches the ground (see Fig. 2, left diagram). At high velocities, the entire wheel might lose ground contact, making it difficult to drive accurately (as happens during RoboCup competitions, [6]).

4 Design of excenter and gears

In order to achieve the movement desired, an excenter is needed. This excenter is held in place by the main axis to which the torque is applied in order to drive the omnidirectional wheel. The first Reuleaux disc's center axis is placed at a distance $\frac{1}{2}d_u$ of the main axis. When the excenter is rotated, the disc's center describes the desired curve (a circle of diameter d_u).

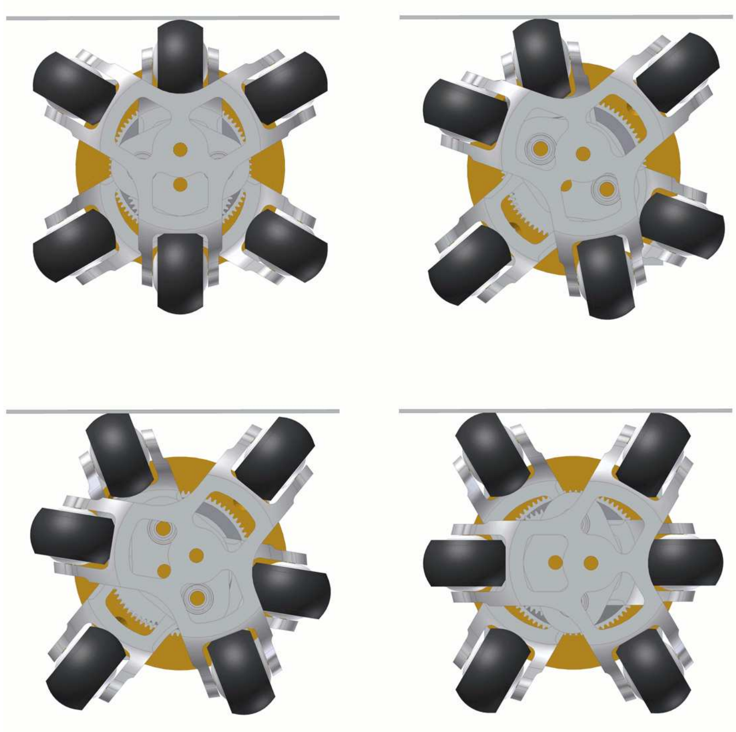

Fig. 7. Rotation of the wheel and relative displacement of the rollers. The wheel starts on the upper left and rolls counterclockwise. The passive roller on the ground loses contact only when the next roller has reached the ground-contact position

Additionally, for each complete rotation around the excenter, the Reulaux disc itself needs to rotate by $-\frac{2}{3}\pi$ around its center. This combined movement is realized by two connected gear wheels. The first gear wheel, with the sprockets on the outside, is fixed to the first Reuleaux disc. Its diameter must be $3 \cdot d_u$. Since this gear wheels moves around the main axis excentrically, a counterweight may be attached to the excenter to compensate the gear wheel's centrifugal force. The second gear wheel, with its sprockets on the inside, is fixed to the robot. Its diameter is be $4 \cdot d_u$. Figure 6 shows the complete configuration.

Now that the first Reuleaux disc describes the desired movement, the second disc needs to be connected to imitate the exact same movement with a phase shift. The phase shift is chosen in such way that the first disc's center travels around the displacement curve through π ahead, with the discs rotated around

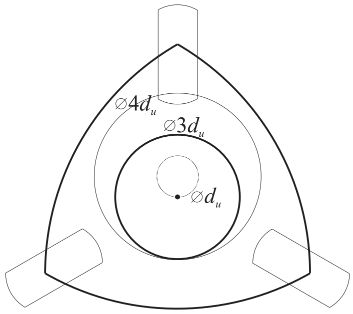

Fig. 8. Gear wheels and their diameter

their centers by $\frac{1}{3}\pi$ to each other (this is explained above by the relation between both movements). Building a gear for this task is rather simple: the second disc's center axis is placed at distance $\frac{1}{2}d_u$ to the main axis, in the opposite direction of the first disc's axis. For the rotation of the second disc itself, the gear is constructed to provide a 1:1 translation from the first to the second disc.

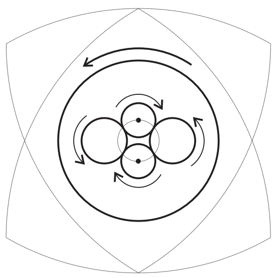

Fig. 9. Assembly and movement of the two Reuleaux discs

5 Conclusions

This paper has presented a new design for an omnidirectional wheel based on Reuleaux triangles. We have shown that even when the complete profile of the wheel mount has "gaps" between the passive rollers, it is indeed possible to

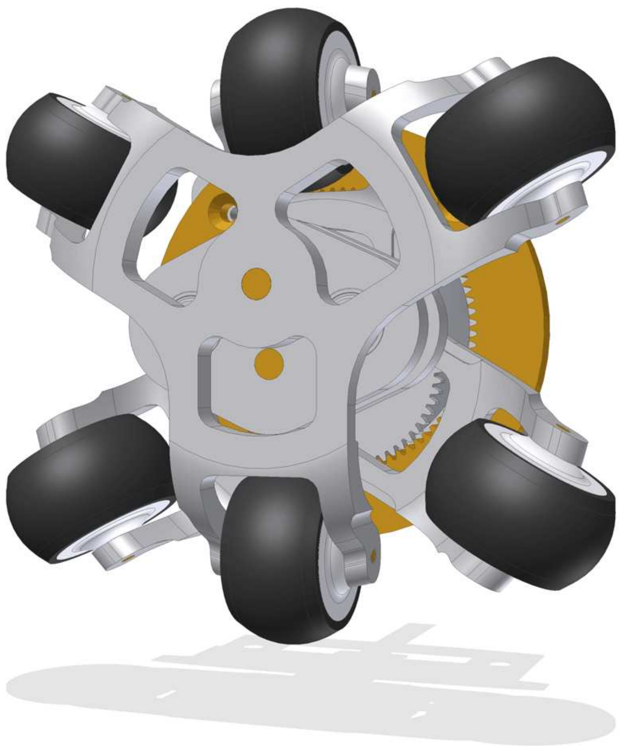

Fig. 10. CAD model of the complete Reuleaux omnidirectional wheel assembly

achieve smooth rotational movement. For this to occur, the wheel center rotates around an excenter. A conventional wheel with profile gaps resembles a polygon and a rotating polygon moves the wheel's center up and down – if nothing is done against that movement.

The wheels used by almost all teams in the small-size league resemble such polygons. Mid-size teams have used also wheels with perpendicular passive rollers and gaps in the wheel profile. The wheels presented in this paper are an alternative solution.

We are aware that the wheel design described in this paper is more complex than other omnidirectional wheels and requires more mechanical parts. Nevertheless, the wheel presented here has some interesting theoretical properties. This design allows a robot (or other vehicle) highly precise and controllable omnidirectional driving, even at high speeds. Wheel grip is optimal because driving is vibration free, and the individual passive wheels have contact with the ground over a long time. Modelling Mecano wheels is extremely complex (because of the angle at which the passive rollers are placed) [7]. Kinematic modeling of the Reuleaux wheels is much simpler.

Fig. 10 shows a ready-to-build sample design, including all the components described above. The wheel has been simulated by animation in a CAD system. A prototype of this wheel will be exhibited at the RoboCup 2006 competition in Bremen. It is a new attempt at reinventing the wheel [8], right, but the omnidirectional wheel.

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