An omnidirectional wheel based on Reuleaux-triangles

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Abstract

Omnidirectional wheels have been used in robotics for several years now. They offer a robot a high degree of freedom in movement. The available kinds of omnidirectional are known to have several disadvantages, such as a lack of wheel grip with the ground over which the robot moves. The faster a robot moves, the more difficult it becomes to steer it to a desired position. Path prediction algorithms are the main method used to avoid these problems nowadays. This paper gives a purely mechanical solution to improve controllability and efficiency of omnidirectional driving, presenting a new wheel design based on Reuleaux-triangles.

Reuleaux-triangles

A so-called Reuleaux-triangle (named by the German engineer Franz Reuleaux) is a geometric shape of constant width, meaning that two parallel tangents touching the shape's outside always have the same distance, independent of the shape's orientation. The most simple shape of constant width is a circle. But also other geometric figures can be extended to have constant width. Figure 1 shows how a Reuleaux triangle is constructed. Based on an equilateral triangle with side length l, constant width is achieved by adding circular arcs with radius l to each of the triangle's sides, with their centers placed in the corner opposing the respective side:

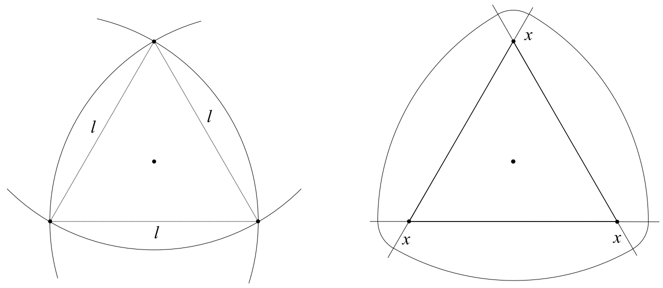

Figure 1

Also, the shape's volume may be increased by extending the side beyond the corners by length x. The arc radii extend to $l + x$, and the gaps that result outside the original triangle are closed by circular arcs with radius x. In the following, the mentioned arcs will be referred to as l_x-section (the smaller arcs of radius x) and l_r-section (the larger arcs).

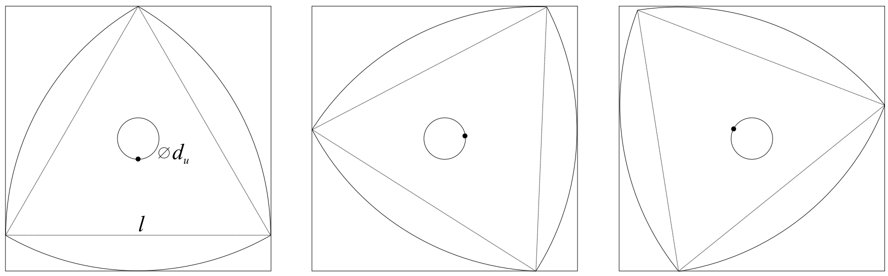

Figure 2

The described characteristics of the Reuleaux-triangle, which we have just constructed, enables the shape to be rotated inside an almost square shape. Merely the corners are rounded. Notice that the triangle's center *does not* remain in the same place while rotating the triangle, but travels around a curve that consists of four ellipse arcs. For the wheel design that is presented in this paper, this curve is approximated by a circle. The circle's diameter d_u can be obtained by the following formula:

$$d_u = l \cdot (\frac{4}{3} \cdot \cos(30°) - 1)$$

This diameter always *remains the same* for a given l, independent of the chosen extension of length x. Therefore, the principal of movement can be generalised from simple Reuleaux-triangles with $x := 0$ to extended triangles, which is important since the wheel design presented here makes use of the extended variant of Reuleaux-triangles.

Wheel design

The omnidirectional wheel proposed in this paper basically consists of the following parts:

1. Two discs based on Reuleaux-triangles, each carrying three passive wheels
2. A gear connecting these discs allowing rotational transmission between these two discs
3. An eccentric that holds both discs and the gear.

Reuleaux discs

A Reuleaux-triangle of enhanced volume is constructed to form the shape of the component which holds three of the six passive wheels that are used with this omnidirectional wheel. The passive wheels' profile needs to be of a specific shape, which is determined by the l_x-sections described above. Spaces remain at the l_r-sections, except for small appendices.

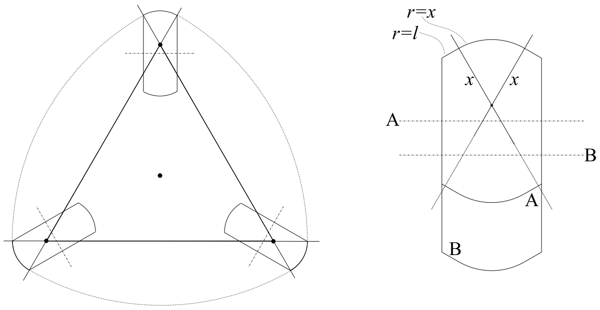

Figure 3

These appendices prevent the passive wheels to lose contact to the ground while another passive wheel has not reached the ground yet. The path described by these passive wheels will be explained below.

The passive wheel is built by using the mentioned shape as the profile of a rotational body. Note that the position of the center axis of this rotational body (which is actually the passive wheel's axis) may be chosen arbitrarily. Thus it is possible to construct passive wheels of different sizes.

Principal of movement

Before describing the construction of the remaining parts in this wheel design, it is necessary for the reader to understand the principal of this omnidirectional wheel's functionality.

As described above, a Reuleaux-triangle can be rotated inside a square. The square's sides are touched alternately by the triangle's corners and the circular arcs (respectively, by the l_x-section and the l_r-section). Imagine one of the square's sides to be the ground over which the presented Reuleaux discs roll, with the passive wheels touching the ground (remember that the passive wheels' profile matches the curve that describes the l_x-section). Contact with the ground is given only half of the time, for when the l_x-section loses contact with one side, the l_r-section reaches this side. The l_r-section is *not* modeled in the Reuleaux discs.

To guarantee ground contact the entire time, a second Reuleaux disc is added to the wheel design, describing the exact same path of movement with the difference that it is phase shifted. The phase shift corresponds to exactly one half of a rotation of the Reuleaux-triangles' centers around the displacement path of diameter d_u.

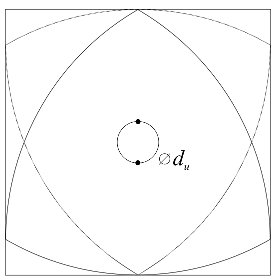

Figure 4

This ensures that the Reuleaux discs' passive wheels always touch the ground, alternating from one disc to the other. A simulation of a working omnidirectional wheel following this principal is shown in figure 5. The advantage of this wheel design becomes clear immediately: *The passive wheels describe a purely linear path along the ground, preventing any up- and down-movement.* Omnidirectional wheels as yet are rather n-gon shaped, which leads to a movement where the robot is lifted up every time a passive wheel passes the ground. At high velocities, the entire wheel might lose ground contact, making it difficult to precisely drive it.

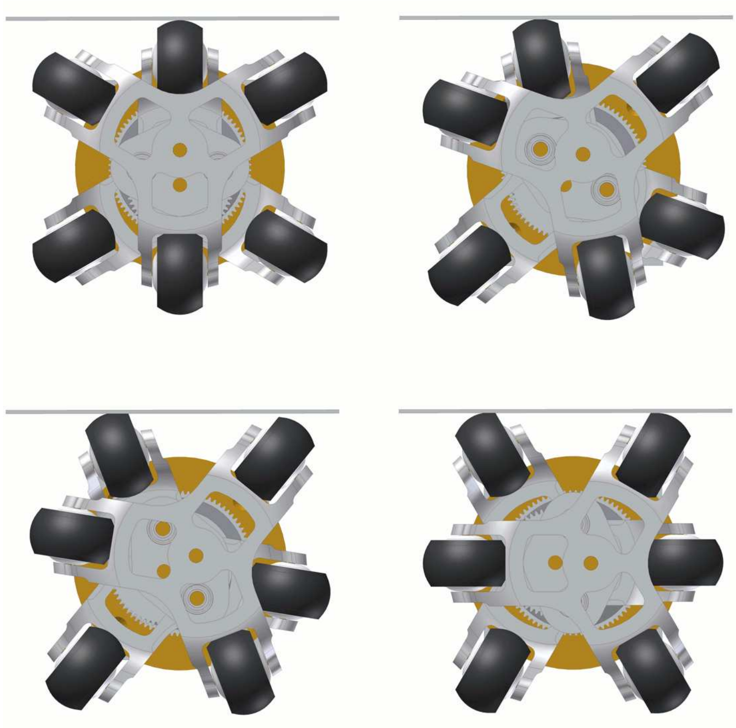

Figure 5

Design of eccentric and gear

In order to achieve the movement desired, an eccentric is needed. This eccentric is held by the main axis to which the torque is applied, driving the omnidirectional wheel. The first Reuleaux disc's center axis is placed at distance $\frac{1}{2}d_u$ to the main axis. When the eccentric is rotated, the disc's center describes the desired displacement curve (a circle of diameter d_u).

Additionally, for each complete rotation around the eccentric (by 2π), the Reulaux disc itself needs to rotate by $-\frac{2}{3}\pi$ around its center. This combined movement is realized by two connected gear wheels. The first gear wheel, with the sprockets on its outside, is statically connected to the first Reuleaux disc. Its diameter must be $3 \cdot d_u$. Since this gear wheels moves around the main axis eccentrically, a counterweight may be attached to the eccentric, compensating the gear wheel's centrifugal force. The second gear wheel, with its sprockets on the inside, is static on the robot. Its diameter is to be $4 \cdot d_u$. Figure 6 shows the complete configuration.

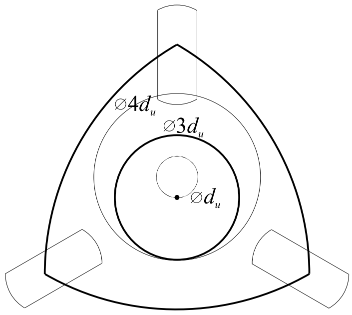

Figure 6

Now that the first Reuleaux disc describes the wanted movement, the second disc needs to be connected to imitate the exact same movement with a phase shift. The phase shift is chosen in such way that the first disc's center travels around the displacement curve through π ahead, with the discs rotated around their centers by $\frac{1}{3}\pi$ to each other (this is explained above by the relation between both movements). Building a gear for this task is rather simple: the second disc's center axis is placed at distance $\frac{1}{2}d_u$ to the main axis, in the opposite direction of the first disc's axis. For the rotation of the second disc itself, the gear is constructed to establish a 1:1 translation from the first to the second disc.

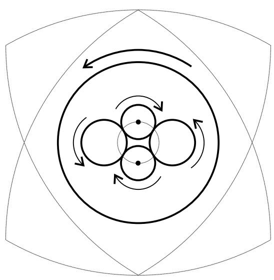

Figure 7

Conclusion

The wheel design presented in this paper is more complex than other omnidirectional wheels, requiring a larger amount of mechanical parts. Nevertheless, the advantages against other solutions are obvious. This design allows a robot (or other vehicle) highly precise and controllable omnidirectional driving, even at high speeds, and still can be constructed to be very robust. Wheel grip is optimal because driving is vibration free, and the individual passive wheels have contact with the ground over a long time. The following figure shows a ready-to-build sample design, including all the components described.

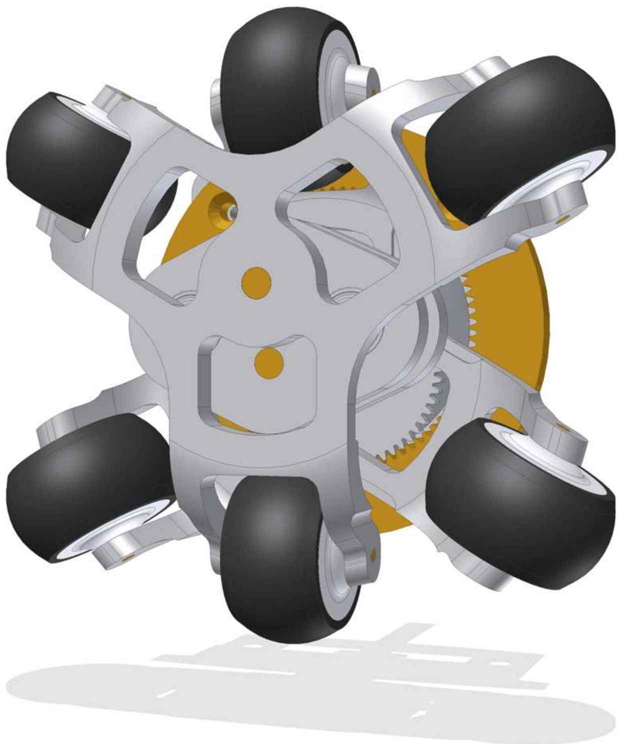

Figure 8